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Splitting of Three Nearly Mass-Degenerate Neutrinos

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Abstract

Assuming the canonical seesaw mechanism together with an SO(3) family symmetry for leptons, broken only by the charged-lepton masses, I show that the three neutrinos of Majorana mass m_0 are split radiatively in two loops by a maximum finite calculable amount of order $10^{-9}\ m_0$. This is very suitable for dark matter and vacuum solar neutrino oscillations. I also discuss how atmospheric neutrino oscillations can be incorporated.

There are now a number of experiments [1, 2, 3] which have varying degrees of evidence for neutrino oscillations. Their implication is that neutrinos must have mass, but since only the differences of the squares of neutrino masses are relevant in these observations, an intriguing possibility exists that each neutrino mass is actually about the same, say of order 1 eV, so as to account for part of the dark matter of the universe [4]. If so, the theoretical challenge is to understand why neutrinos are nearly degenerate in mass <u>and</u> why their splittings are so small.

In the context of the canonical seesaw mechanism [5] for small Majorana neutrino masses, a common mass m_0 of order 1 eV may be obtained with the imposition of an SO(3) family symmetry [6]. Since the charged-lepton masses break the above symmetry, the three neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ are then split radiatively in two loops [7] by a maximum finite calculable amount of order $10^{-9}\ m_0$. This is a consequence of the fact that a Majorana neutrino mass term in the minimal standard model comes from an effective operator of dimension five [8, 9]. Hence vacuum solar neutrino oscillations with $\Delta m^2 \sim 10^{-10}$ eV^2 are natural in this scenario. With further assumptions, a specific model is presented in the following which has maximal mixing for solar and atmospheric neutrino oscillations. It is also consistent with the absence of neutrinoless double beta decay [10].

Consider three standard-model lepton doublets $(\nu_i, l_i)_L$ and three heavy neutrino singlets N_{iR}, where the subscript i refers to the $(+, 0, -)$ components of an SO(3) triplet. Let $\Phi = (\phi^+, \phi^0)$ be the usual Higgs doublet, then the SO(3)-invariant term linking $(\nu_i, l_i)_L$ to N_{iR} is

$$f\left[(\bar\nu_+ N_+ + \bar\nu_0 N_0 + \bar\nu_- N_-)\,\bar\phi^0 - \left(\bar l_+ N_+ + \bar l_0 N_0 + \bar l_- N_-\right)\phi^-\right], \tag{1}$$

and the SO(3)-invariant Majorana mass term for N_{iR} is

$$M\,(2N_+N_- - N_0N_0)\,. \tag{2}$$

As ϕ^0 acquires a nonzero vacuum expectation value $\langle\phi^0\rangle = v$, the 6×6 mass matrix spanning

$(\bar{\nu}_+, \bar{\nu}_-, \bar{\nu}_0, N_+, N_-, N_0)$ is given by

$$\mathcal{M}_{\nu,N} = \left[\begin{array}{cccccc} 0 & 0 & 0 & m_D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_D \\ m_D & 0 & 0 & 0 & M & 0 \\ 0 & m_D & 0 & M & 0 & 0 \\ 0 & 0 & m_D & 0 & 0 & -M \end{array} \right], \tag{3}$$

where $m_D = fv$. Invoking the well-known seesaw mechanism [5], the 3×3 mass matrix spanning (ν_+, ν_-, ν_0) is then

$$\mathcal{M}_\nu = \left[\begin{array}{ccc} 0 & -m_0 & 0 \\ -m_0 & 0 & 0 \\ 0 & 0 & m_0 \end{array} \right], \tag{4}$$

where $m_0 = m_D^2/M$.

The physical identites of ν_i depend on the charged-lepton mass matrix which breaks the assumed SO(3) family symmetry. As a working hypothesis, consider the following basis:

$$l_+ = e, \quad l_- = c\mu + s\tau, \quad \text{and} \quad l_0 = c\tau - s\mu, \tag{5}$$

where $c = \cos\theta$ and $s = \sin\theta$. The justification for it will come later. Furthermore, let there be an additional mass term m_1 for the state $c'\nu_0 + s'(\nu_+ - \nu_-)/\sqrt{2}$, where $c' = \cos\theta'$ and $s' = \sin\theta'$. This is equivalent to breaking the SO(3) symmetry of Eq. (4) explicitly at tree level. It will be shown later where m_1 comes from and how it is related to atmospheric neutrino oscillations. The generic statement that the ν_i's of Eq. (4) are naturally split by a maximum amount of order $10^{-9}\ m_0$ is independent of the above details. However, they are required for a specific model which explains the present data on both solar and atmospheric neutrino oscillations as well as hot dark matter and the absence of neutrinoless double beta decay.

In the minimal standard model, any neutrino mass must come from the effective operator

[8]

$$\Lambda^{-1}\phi^0\phi^0\nu_i\nu_j, \qquad (6)$$

where Λ is a large effective mass. In the canonical seesaw mechanism [5], neutrino masses are generated at tree level [9] and may all be different. However, in the presence of an SO(3) family symmetry which is broken only by charged-lepton masses, the radiative splitting is now guaranteed to be finite and calculable. The effective low-energy theory is exactly the minimal standard model extended to include a common mass m_0 for all three neutrinos. The specific mechanism for their splitting is the exchange of two W bosons in two loops [7], as shown in Fig. 1. Since m_τ is the largest charged-lepton mass by far, the breaking is along the τ direction in lepton space. (The extra mass term m_1 breaks the SO(3) symmetry explicitly along a different direction and will be considered later.)

The two-loop diagram of Fig. 1 may be evaluated using Eqs. (3) to (5). The generic structure of the double integral is [7, 11]

$$g^4 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{p^2-m_W^2} \frac{1}{q^2-m_W^2} \frac{1}{p^2-m_{l_i}^2} \frac{1}{q^2-m_{l_j}^2} \frac{p\cdot q}{(p+q)^2-m_0^2} \frac{m_D^2 M}{(p+q)^2-M^2}. \qquad (7)$$

By dimension analysis, it is clear that the above is proportional to $m_D^2/M = m_0$. Expanding in powers of m_l^2/m_W^2, it is also clear that there is a universal contribution to m_0 which one can disregard, and the splitting among the three neutrinos is determined by a term proportional to m_τ^2/m_W^2. (Contributions from ϕW and $\phi\phi$ exchanges are negligible because they are at most of order m_τ^4/m_W^4.) Replacing one of the factors involving m_l in Eq. (7), say $(p^2-m_\tau^2)^{-1}$, with m_τ^2/p^4, the resulting integral can be evaluated exactly in the limit $m_0^2 << m_l^2 << m_W^2 << M^2$:

$$I = \frac{g^4}{256\pi^4}\frac{m_\tau^2}{M_W^2}\left(\frac{\pi^2}{6}-\frac{1}{2}\right)m_0 = 3.6\times 10^{-9}\ m_0. \qquad (8)$$

Consequently, $\mathcal{M}_\nu$ of Eq. (4) becomes

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & -m_0 - s^2 I & -scI \\ -m_0 - s^2 I & 0 & scI \\ -scI & scI & m_0 + 2c^2 I \end{bmatrix}, \tag{9}$$

where m_0 has been redefined to absorb the universal radiative contribution mentioned earlier.

Whereas the eigenvalues of Eq. (4) are $-m_0$, m_0, and m_0, corresponding to the eigenstates $(\nu_+ + \nu_-)/\sqrt{2}$, $(\nu_+ - \nu_-)/\sqrt{2}$, and ν_0, those of Eq. (9) are

$$-m_0 - s^2 I, \quad m_0, \quad \text{and} \quad m_0 + (1 + c^2) I, \tag{10}$$

corresponding to the eigenstates

$$\frac{\nu_+ + \nu_-}{\sqrt{2}}, \quad \frac{c\nu_+ - c\nu_- + s\nu_0}{\sqrt{1+c^2}}, \quad \text{and} \quad \frac{-s\nu_+ + s\nu_- + 2c\nu_0}{\sqrt{2(1+c^2)}}. \tag{11}$$

For positive m_0, the eigenvalue $-m_0 - s^2 I$ is negative. However, as is well-known, it becomes positive under a γ_5 rotation of its corresponding eigenstate. Comparing Eq. (5) with Eq. (11) and using Eq. (10), the probability of ν_e oscillations in vacuum is given by

$$\begin{aligned} P(\nu_e \to \nu_e) \quad &= \quad \frac{1+3c^4}{2(1+c^2)^2} + \frac{c^2}{1+c^2} \cos\left(\frac{s^2 \Delta m_0^2 t}{2E}\right) + \frac{s^2}{2(1+c^2)} \cos\left(\frac{2c^2 \Delta m_0^2 t}{2E}\right) \\ &+ \frac{s^2 c^2}{(1+c^2)^2} \cos\left(\frac{(1+c^2)\Delta m_0^2 t}{2E}\right), \end{aligned} \tag{12}$$

where

$$\Delta m_0^2 = 2 m_0 I = 7.2 \times 10^{-9}\ m_0^2. \tag{13}$$

For $m_0 = 2$ eV and $s^2 = 0.01$, solar neutrino oscillations are then interpreted here as mostly $\nu_e \to \nu_\mu$ with $\sin^2 2\theta \simeq 1$ and $\Delta m^2 \simeq 3 \times 10^{-10}$ eV^2, in good agreement [12] with data [1].

The choice of basis given by Eq. (5) corresponds to the following charged-lepton mass matrix linking $(\bar{l}_+, \bar{l}_-, \bar{l}_0)_L$ with $(e, \mu, \tau)_R$:

$$\mathcal{M}_l = \begin{bmatrix} m_e & 0 & 0 \\ 0 & cm_\mu & sm_\tau \\ 0 & -sm_\mu & cm_\tau \end{bmatrix}. \tag{14}$$

This is based on essentially just one assumption, *i.e.* that the $\nu_e - \nu_e$ entry of the neutrino mass matrix [Eqs. (4) and (9)] is in fact zero. Neutrinoless double beta decay is then guaranteed to be absent in lowest order despite the fact that m_0 may be of order 1 eV. Note that in general, one can always choose the l_{iR} basis so that the two zeros appear in the first row of $\mathcal{M}_l$. After that, one needs to make the assumption that $l_+ = e$ to have the two zeros in the first column of $\mathcal{M}_l$. The remaining 2×2 submatrix is then automatically as given.

In addition to $\mathcal{M}_l$ which breaks the SO(3) family symmetry explicitly, consider now the possible origin of m_1 for the state $c'\nu_0 + s'(\nu_+ - \nu_-)/\sqrt{2}$. Let there be an extra heavy neutrino singlet N' and an extra Higgs doublet Φ', both of which are odd under a new discrete Z_2 symmetry. In that case, the term

$$f' \left[(c'\bar{\nu}_0 + s'(\bar{\nu}_+ - \bar{\nu}_-)/\sqrt{2}) N' \bar{\phi}'^0 - (c'\bar{l}_0 + s'(\bar{l}_+ - \bar{l}_-)/\sqrt{2}) N' \phi'^- \right] + H.c. \tag{15}$$

also breaks the SO(3) family symmetry explicitly and

$$m_1 = \frac{(f'v')^2}{M'}, \tag{16}$$

where M' is the Majorana mass of N' and $v' = \langle \phi'^0 \rangle$. Now v' may be naturally small compared to v if Φ' is heavy [13]. From the terms $m'^2 \Phi'^\dagger \Phi'$ and $\mu^2 (\Phi'^\dagger \Phi + \Phi^\dagger \Phi')$ in the Higgs potential, it can easily be shown that

$$v' \simeq \frac{-\mu^2 v}{m'^2}. \tag{17}$$

Since the μ^2 term breaks the discrete Z_2 symmetry softly, $v'/v \sim 10^{-2}$ is a reasonable assumption. For $M' \sim M$ and $f' \sim f$, a value of $m_1/m_0 = 5 \times 10^{-4}$ is thus very natural. Hence atmospheric neutrino oscillations [14] may occur between ν_μ and ν_τ with

$$\Delta m^2_{\rm atm} \simeq (m_0 + m_1)^2 - m_0^2 \simeq 2 m_0 m_1 \simeq 4 \times 10^{-3} \text{ eV}^2 \tag{18}$$

if $m_0 = 2$ eV, and the mixing angle is θ if θ' is small, which turns out to be necessary if solar neutrino oscillations are to be accommodated at the same time, as shown below.

Inserting m_1 into $\mathcal{M}_\nu$ of Eq. (9), we find that the mass eigenstates are now

$$\frac{\nu_+ + \nu_-}{\sqrt{2}}, \quad \frac{c'(\nu_+ - \nu_-)}{\sqrt{2}} - s'\nu_0, \quad \frac{s'(\nu_+ - \nu_-)}{\sqrt{2}} + c'\nu_0, \tag{19}$$

with eigenvalues

$$-m_0 - s^2 I, \quad m_0 + (c'^2 s^2 + 2s'^2 c^2 + 2\sqrt{2}s'c'sc)I, \quad m_0 + m_1. \tag{20}$$

Hence

$$\begin{aligned} \Delta m^2_{\rm sol} &\simeq [m_0 + (c'^2 s^2 + 2s'^2 c^2 + 2\sqrt{2}s'c'sc)I]^2 - [m_0 + s^2 I]^2 \\ &\simeq 2m_0[2\sqrt{2}s'c'sc + s'^2(2 - 3s^2)]I \;\simeq\; 4\sqrt{2}scs'm_0 I \end{aligned} \tag{21}$$

if $s' << 1$. Let $s = c = 1/\sqrt{2}$ for maximal mixing in atmospheric neutrino oscillations (which is not required by this model, but an additional ssumption), then

$$\Delta m^2_{\rm sol} \simeq 4 \times 10^{-10} \text{ eV}^2 \tag{22}$$

if $s' = 0.01$ and $m_0 = 2$ eV.

If m_1 is absent, then Eq. (12) governs solar neutrino oscillations, and there is no explanation of atmospheric neutrino oscillations. If m_1 is present, then atmospheric neutrino oscillations are automatically accounted for, but now s' has to be small to explain solar neutrino oscillations. Hence m_1 should correspond dominantly but not completely to $\nu_0 = c\nu_\tau - s\nu_\mu$. In fact, although it is assumed that ν_0 mixes only with $(\nu_+ - \nu_-)/\sqrt{2}$, the above conclusion will not change if there is also mixing with $(\nu_+ + \nu_-)/\sqrt{2}$ as long as it is small.

Let the final neutrino mass matrix be rewritten in the basis $(\nu_e, \nu_\mu, \nu_\tau)_L$:

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & -c & -s \\ -c & s^2 & -sc \\ -s & -sc & c^2 \end{bmatrix} m_0 \; + \; \begin{bmatrix} 0 & 0 & -s \\ 0 & 0 & -sc \\ -s & -sc & 2c^2 \end{bmatrix} I \; + \tag{23}$$

$$\begin{bmatrix} s'^2/2 & -s'(c's/\sqrt{2}+s'c/2) & s'(c'c/\sqrt{2}-s's/2) \\ -s'(c's/\sqrt{2}+s'c/2) & (c's+s'c/\sqrt{2})^2 & -(c'c-s's/\sqrt{2})(c's+s'c/\sqrt{2}) \\ s'(c'c/\sqrt{2}-s's/2) & -(c'c-s's/\sqrt{2})(c's+s'c/\sqrt{2}) & (c'c-s's/\sqrt{2})^2 \end{bmatrix} m_1.$$

The form of the dominant m_0 term is exactly the one advocated recently [15] if $s = c = 1/\sqrt{2}$ is assumed. The transformation matrix between $\nu_{e,\mu,\tau}$ and the mass eigenstates $\nu_{1,2,3}$ of Eq. (19) is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & c'/\sqrt{2} & s'/\sqrt{2} \\ c/\sqrt{2} & s's-c'c/\sqrt{2} & -c's-s'c/\sqrt{2} \\ s/\sqrt{2} & -s'c-c's/\sqrt{2} & c'c-s's/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{24}$$

In the limit $s' = 0$ and $s = c = 1/\sqrt{2}$, it reduces to

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/2 & -1/2 & -1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{25}$$

which shows clearly that both $\nu_e \to \nu_e$ and $\nu_\mu \to \nu_\tau$ oscillations are maximal. Note that the $\nu_e - \nu_e$ entry of Eq. (23) is now $s'^2 m_1/2$, *i.e.* of order 10^{-8} eV, which is certainly still negligible for neutrinoless double beta decay.

In conclusion, the idea of nearly mass-degenerate neutrinos [6, 15, 16] of a few eV should not be overlooked since they may well be the hot dark matter of the universe [4]. A simple and realistic model has been proposed, where their splittings are finite calculable radiative corrections and are very suitable for vacuum solar neutrino oscillations. To allow for atmospheric neutrino oscillations as well, additional explicit breaking of the assumed SO(3) family symmetry may be required. An alternative explanation is to have flavor-changing neutrino interactions [17], but that is subject to other serious experimental constraints [18].

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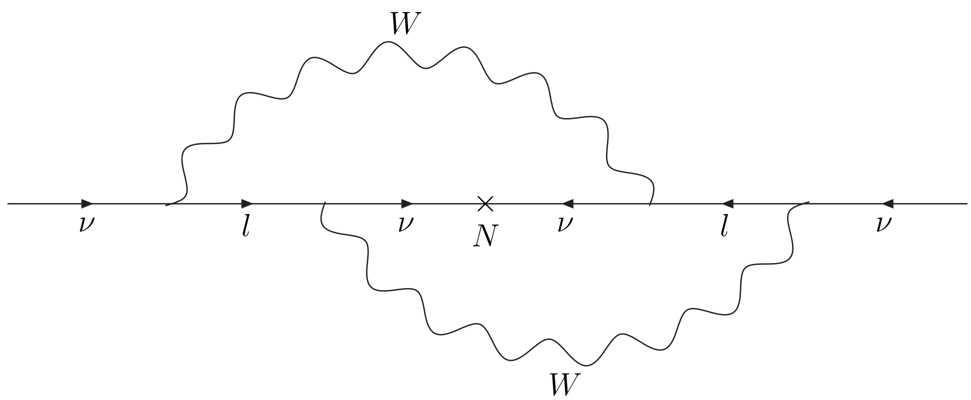

Fig. 1. Two-loop radiative breaking of neutrino mass degeneracy.